Reg. No. : | | | | | | | | | | | |

Question Paper Code : 41320

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fifth Semester

Computer Science and Engineering

MA 6566 – DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Define proposition.
2. Give the symbolic form of "Some men are giant".
3. Define Pigeon hole principle.
4. How many permutations can be made out of letter or word 'COMPUTER' ?
5. Show that there does not exist a graph with 5 vertices with degrees 1, 3, 4, 2, 3 respectively.
6. Define Hamiltonian path.
7. Define semi group.
8. Prove that in a group idempotent law is true only for identity element.
9. Let A = {1, 2, 5, 10} with the relation divides. Draw the Hasse diagram.
10. Prove that a lattice with five elements is not a Boolean algebra.

PART – B (5×16=80 Marks)

11. a) i) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$, without using truth table. (8)

ii) Show that using Rule C.P, $\neg P \vee Q$, $\neg Q \vee R$, $R \rightarrow S \Rightarrow P \rightarrow S$ (8)

(OR)

b) i) Find the PCNF of $(P \vee R) \wedge (P \vee \neg Q)$ Also find its PDNF, without using truth table. (8)

ii) Show that $(\forall x)\,[P(x) \vee Q(x)] \Rightarrow (\forall x)\,P(x) \vee (\exists x)\,Q(x)$. (8)

12. a) i) Prove that $n^3 - n$ is divisible by 3 for $n \geq 1$ (8)

ii) Solve $G(k) - 7G(k-1) + 10G(k-2) = 8k + 6$. (8)

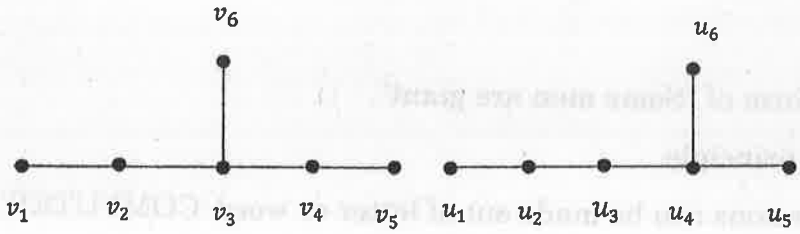
(OR)

b) i) Find the numbers between 1 to 250 that are not divisible by any of the integers 2 or 3 or 5 or 7. (8)

ii) Solve using generating functions : $S(n) + 3S(n-1) - 4S(n-2) = 0$; $n \geq 2$ given $S(0) = 3$, $S(1) = -2$. (8)

13. a) i) State and prove Hand shaking theorem. Hence prove that for any simple graph G with n vertices, the number of edges of G is less than or equal to $\frac{n(n-1)}{2}$. (8)

ii) Establish the isomorphism of the following pairs of graphs. (8)



(OR)

b) i) Prove that a graph G is disconnected if and only if its vertex set V can be partitioned into two non-empty, disjoint subsets V_1 and V_2 such that there exists no edge in G whose one end vertex is in subset V_1 and the other in subset V_2. (8)

ii) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of even degree. (8)

14. a) i) Show that $(Q^+, *)$ is an abelian group, where * is defined by

$$a * b = \frac{ab}{2}, \forall a, b \in Q^+$$ (8)

ii) Prove that kernel of a homomorphism is a normal subgroup of G. (8)

(OR)

b) i) Prove that intersection of two normal subgroups of a group G is again a normal subgroup of G. (8)

ii) Let G be a finite group and H be a subgroup of G. Then prove that order of H divides order of G. (8)

15. a) i) Show that $(N, \leq)$ is a partially ordered set, where N is the set of all positive integers and $\leq$ is a relation defined by $m \leq n$ if and only if $n - m$ is a non-negative integer. **(8)**

ii) In a complemented and distributive lattice, prove that complement of each element is unique. **(8)**

(OR)

b) i) Let $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$ with a relation $x \leq y$ if and only if x divides y. Find :

i) All lower bounds of 10 and 15

ii) GLB of 10 and 15

iii) All upper bound are 10 and 15

iv) LUB of 10 and 15

v) Draw the Hasse diagram for D_{30}. **(8)**

ii) Let $(L, , \vee, \wedge, \leq)$ be a distributive lattice and $a, b, \in L$ if $a \wedge b = a \wedge c$ and $a \vee b = a \vee c$. Then show that $b = c$. **(8)**

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15. a) i) Show that (N, ≤) is a partially ordered set, where N is the set of all positive integers and ≤ is a relation defined by m ≤ n if and only if n – m is a non-negative integer. (6)

ii) In a complemented and distributive lattice, prove that complement of each element is unique. (8)

(OR)

b) i) Let D_{30} = {1, 2, 3, 5, 6, 10, 15, 30} with a relation x ≤ y if and only if x divides y. Find:

i) All lower bounds of 10 and 15

ii) GLB of 10 and 15

iii) All upper bound are 10 and 15

iv) LUB of 10 and 15

v) Draw the Hasse diagram for D_{30}. (8)

ii) Let (L, ∨, ∧, ≤) be a distributive lattice and a, b, c ∈ L. If a ∧ b = a ∧ c and a ∨ b = a ∨ c. Then show that b = c. (6)